General Certificate of Education
Advanced Level Examination
January 2012

Mathematics MPC3

Unit Pure Core 3

Friday 20 January 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 (a) Use Simpson's rule with 7 ordinates (6 strips) to find an estimate for $\int_0^3 4^x \, dx$. *(4 marks)*

(b) A curve is defined by the equation $y = 4^x$. The curve intersects the line $y = 8 - 2x$ at a single point where $x = \alpha$.

(i) Show that α lies between 1.2 and 1.3. *(2 marks)*

(ii) The equation $4^x = 8 - 2x$ can be rearranged into the form $x = \dfrac{\ln(8 - 2x)}{\ln 4}$.

Use the iterative formula $x_{n+1} = \dfrac{\ln(8 - 2x_n)}{\ln 4}$ with $x_1 = 1.2$ to find the values of x_2 and x_3, giving your answers to three decimal places. *(2 marks)*

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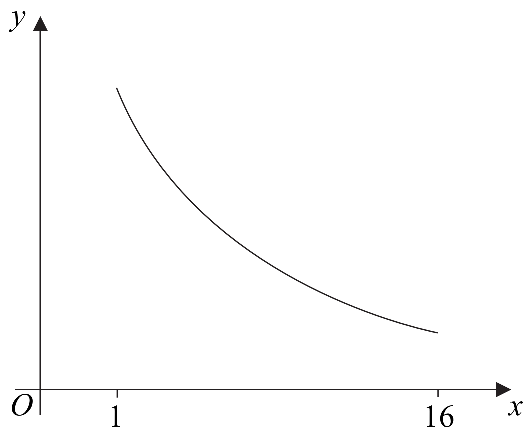
2 The curve with equation $y = \dfrac{63}{4x - 1}$ is sketched below for $1 \leqslant x \leqslant 16$.



The function f is defined by $\mathrm{f}(x) = \dfrac{63}{4x - 1}$ for $1 \leqslant x \leqslant 16$.

(a) Find the range of f. *(2 marks)*

(b) The inverse of f is f^{-1}.

(i) Find $\mathrm{f}^{-1}(x)$. *(3 marks)*

(ii) Solve the equation $\mathrm{f}^{-1}(x) = 1$. *(2 marks)*

(c) The function g is defined by $\mathrm{g}(x) = x^2$ for $-4 \leqslant x \leqslant -1$.

(i) Write down an expression for $\mathrm{fg}(x)$. *(1 mark)*

(ii) Solve the equation $\mathrm{fg}(x) = 1$. *(3 marks)*

3 (a) Given that $y = 4x^3 - 6x + 1$, find $\frac{dy}{dx}$. *(1 mark)*

(b) Hence find $\int_2^3 \frac{2x^2 - 1}{4x^3 - 6x + 1}\,dx$, giving your answer in the form $p \ln q$, where p and q are rational numbers. *(5 marks)*

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4 (a) By using a suitable trigonometrical identity, solve the equation

$$\tan^2\theta = 3(3 - \sec\theta)$$

giving all solutions to the nearest 0.1° in the interval $0° < \theta < 360°$. *(6 marks)*

(b) Hence solve the equation

$$\tan^2(4x - 10°) = 3[3 - \sec(4x - 10°)]$$

giving all solutions to the nearest 0.1° in the interval $0° < x < 90°$. *(3 marks)*

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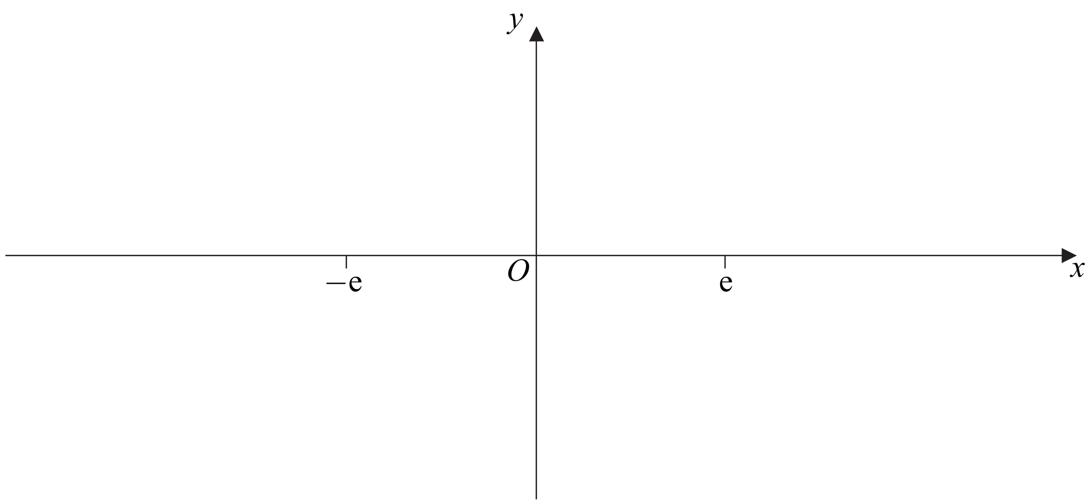
5 (a) Describe a sequence of two geometrical transformations that maps the graph of $y = \ln x$ onto the graph of $y = 4\ln(x - e)$. *(4 marks)*

(b) Sketch, on the axes given below, the graph of $y = |4\ln(x - e)|$, indicating the exact value of the x-coordinate where the curve meets the x-axis. *(3 marks)*

(c) (i) Solve the equation $|4\ln(x - e)| = 4$. *(3 marks)*

(ii) Hence, or otherwise, solve the inequality $|4\ln(x - e)| \geqslant 4$. *(3 marks)*



6 (a) Given that $x = \dfrac{1}{\sin\theta}$, use the quotient rule to show that $\dfrac{dx}{d\theta} = -\operatorname{cosec}\theta\cot\theta$.

(3 marks)

(b) Use the substitution $x = \operatorname{cosec}\theta$ to find $\displaystyle\int_{\sqrt{2}}^{2} \frac{1}{x^2\sqrt{x^2-1}}\,dx$, giving your answer to three significant figures.

(9 marks)

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7 (a) A curve has equation $y = x^2 e^{-\frac{x}{4}}$.

Show that the curve has exactly two stationary points and find the exact values of their coordinates.

(7 marks)

(b) (i) Use integration by parts twice to find the exact value of $\displaystyle\int_0^4 x^2 e^{-\frac{x}{4}}\,dx$.

(7 marks)

(ii) The region bounded by the curve $y = 3xe^{-\frac{x}{8}}$, the x-axis from 0 to 4 and the line $x = 4$ is rotated through 360° about the x-axis to form a solid.

Use your answer to part **(b)(i)** to find the exact value of the volume of the solid generated.

(2 marks)